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# Studying Indirect Violation of CP, T and CPT in a $B$ -factory

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## ABSTRACT

In this work we analyze the observable asymmetries one can build from entangled  $B$ -meson states, in order to extract information on the parameters  $\varepsilon$  and  $\delta$  which govern indirect violation of discrete symmetries. The traditionally proposed observables, based on flavour tags, are not helpful for the study of the  $B_d$ -system, where the tiny value of  $\Delta\Gamma$  clears up such asymmetry effects. Our study makes instead use of CP tags in order to build new asymmetries where the different parameters can be separated out. For this separation, it is decisive to achieve a good time resolution in the measurement of entangled state decays. Nevertheless, even with no temporal information, as would be the case in a symmetric factory, it is still possible to extract some information on the symmetries of the system. We discuss both non-genuine and genuine observables, depending on whether absorptive parts can mimic or not asymmetry effects.

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# 1 Introduction

The concept of indirect violation of discrete symmetries in a neutral meson system makes reference to the non-invariance properties of the effective hamiltonian which governs the time evolution of mesons [1]. Those properties can be analyzed by studying the symmetries in the problem of mixing during the time evolution of meson states, while the possible effects in direct decays must be excluded. In these phenomena of time evolution, T violation is evidenced if, for a given time interval, one detects a difference between the probabilities for  $i \rightarrow f$  and  $f \rightarrow i$ , being  $i$  and  $f$  shortnames for neutral meson states. CPT violation would be seen as a difference between  $i \rightarrow f$  and  $\bar{f} \rightarrow \bar{i}$ .

For the kaon system, this study has been performed by the CP-LEAR experiment [2] from the preparation of definite flavour states  $K^0$ - $\bar{K}^0$ . These tagged mesons evolve in time and their later decay to a semileptonic final state projects them again on a definite flavour state. The study of this *flavour-to-flavour* evolution allows the construction of observables which violate CP and T, or CP and CPT. The results are interpreted in terms of non-invariances of the effective hamiltonian, although some discussion remains [3] on the possible CPT violation in semileptonic decays.

Nevertheless, these T- and CPT-odd observables would be zero, even in presence of T and CPT fundamental violation, if there were no absorptive components in the effective hamiltonian. For the kaon system, the different lifetimes of physical states,  $K_S$  and  $K_L$ , ensures this is not the case. On the contrary, in the case of the  $B_d$ -system, the width difference  $\Delta\Gamma$  between the physical states is expected [4] to be negligible. Then the T- and CPT-odd observables proposed for kaons, which are based on flavour tag, vanish for a  $B$ -meson system with  $\Delta\Gamma = 0$ . But the  $B_d$  entangled states can be used to construct alternative observables which are sensitive to T and CPT independently of the value of  $\Delta\Gamma$  [5].

In this paper we study three different types of observables that can be constructed from the entangled states of  $B_d$  mesons in order to study indirect violation of CP, T and CPT. We separate the observables into two different categories

1. *genuine* asymmetries, characterized by the fact that they are pure symmetry observables, constructed by comparing the probabilities of two conjugated processes, so that the asymmetry vanishes if the relating symmetry is conserved;
2. *non-genuine* asymmetries, which do not correspond to purely conjugated pairs of processes, so that its non-vanishing value can be mimic by the presence of absorptive parts.

Within the first category we can distinguish two types of observables depending on whether they need the support of absorptive parts in order to give non-vanishing asymmetries.

To start with we give in section 2 a brief overview of the formalism and notation used to study the problem of indirect violation of discrete symmetries in the  $B$ -system. Then, in section 3, we will review how to construct the flavour asymmetries, the analogous in the  $B$ -system to those observables measured for kaons. They turn out to be proportional to  $\Delta\Gamma$ ,

and their value is expected to be so small that they will not be useful to study the symmetry properties in the case of the  $B_d$ -system. They constitute the first type of observables we are studying, and belong to the *genuine* category.

In section 4 we construct alternative asymmetries, based on a CP tag. In that way we find CP-, T- and CPT-odd time-dependent asymmetries, whose values do not cancel even for  $\Delta\Gamma = 0$ . This will be the second type of observables in our classification, offering the best chances to study the symmetries in  $B_d$ -mixing. The resulting asymmetries are also *genuine* observables.

The limit of small  $\Delta\Gamma$  causes the time-reversal operation and the exchange of decay products to be equivalent. We exploit this fact in section 5 to build the third kind of asymmetries that involve only the  $J/\Psi K_S$  final state, experimentally easy to identify. These observables are *non-genuine*, since their equivalence to the true T- and CPT-odd asymmetries only holds in the exact limit  $\Delta\Gamma = 0$ .

Therefore in section 6 we will see how the introduction of  $\Delta\Gamma \neq 0$  affects the results in sections 4 and 5, and we will discuss whether it is still possible to extract information on the CP, T and CPT parameters from the non-genuine asymmetries even in presence of fake effects due to absorptive parts.

We emphasize that the time dependence of the observables plays a relevant role in order to separate genuine and fake effects. But it is still possible to extract some limited information with measurements that have no time resolution, as we will discuss in section 7.

Finally, in section 8 we will summarize our conclusions.

## 2 The entangled state of $B$ -mesons

In a  $B$  factory operating at the  $\Upsilon(4S)$  peak, correlated pairs of neutral  $B$ -mesons are produced through the reaction:

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}. \quad (1)$$

In the CM frame, the resulting  $B$ -mesons travel in opposite directions, and each one will evolve with the effective hamiltonian of the neutral mesons system. Due to the intrinsic spin of  $\Upsilon(4S)$ , the so formed state has definite  $L = 1$ . Bose statistics requires the physical  $B^0\bar{B}^0$  state to be symmetric under  $C\mathcal{P}$ , being  $\mathcal{P}$  the operator which permutes the spatial coordinates. Together with this requirement,  $C = -$  implies that  $\mathcal{P} = -$ , so that the initial state may be written as

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |B^0(\vec{k}), \bar{B}^0(-\vec{k})\rangle - |\bar{B}^0(\vec{k}), B^0(-\vec{k})\rangle \right). \quad (2)$$

As a consequence, one can never simultaneously have two identical mesons at both sides of the detector. This permits the performance of a flavour tag: if at  $t = 0$  one of the mesons

decays through a channel which is only allowed for one flavour of the neutral  $B$ , the other meson in the pair must have the opposite flavour at  $t = 0$ . The correlation (2) between both sides of the entangled state holds at any time after the production, until the moment of the first decay.

The entangled  $B - \bar{B}$  state can also be expressed in terms of the CP eigenstates  $|B_{\pm}\rangle \equiv \frac{1}{\sqrt{2}} (|B^0\rangle \pm \text{CP}|B^0\rangle)$  as

$$|i\rangle = \frac{1}{\sqrt{2}} \left( |B_-(\vec{k}), B_+(-\vec{k})\rangle - |B_+(\vec{k}), B_-(-\vec{k})\rangle \right). \quad (3)$$

Thus, if the CP operator is well defined, it is also possible to carry out a CP tag. It is enough for that to have a CP-conserving decay into a definite CP final state, so that its detection allows us to identify the decaying meson as a  $B_+$  or a  $B_-$ . As in the flavour case, it is possible to perform such a tag at any time after the production of the entangled state.

In Ref. [6] we described how the determination of the CP operator is possible and unambiguous to  $\mathcal{O}(\lambda^3)$ , which is sufficient to discuss both CP-conserving and CP-violating amplitudes in the effective hamiltonian for  $B_d$  mesons. Here  $\lambda$  is the flavour-mixing parameter of the CKM matrix [7]. The determination is based on the requirement of CP conservation, to  $\mathcal{O}(\lambda^3)$ , in the  $(sd)$  and  $(bs)$  sectors. To this order, however, CP-violation exists in the  $(bd)$  sector, and it can be classified by referring it to the CP-conserving direction. A  $B_d$  decay that is governed by the couplings of the  $(sd)$  or  $(bs)$  unitarity triangles, or by the  $V_{cd}V_{cb}^*$  side of the  $(bd)$  triangle, will not show any CP violation to  $\mathcal{O}(\lambda^3)$ . We may say that such a channel is free from direct CP violation.

Two complex parameters,  $\varepsilon_{1,2}$ , describe the CP mixing in the physical states

$$\begin{aligned} |B_1\rangle &= \frac{1}{\sqrt{1+|\varepsilon_1|^2}} [|B_+\rangle + \varepsilon_1|B_-\rangle] , \\ |B_2\rangle &= \frac{1}{\sqrt{1+|\varepsilon_2|^2}} [|B_-\rangle + \varepsilon_2|B_+\rangle] . \end{aligned} \quad (4)$$

They are invariant under rephasing of the meson states, and physical when the CP operator is well defined [8].

In case of CPT conservation,  $\varepsilon_1 = \varepsilon_2$ . There is another pair of parameters,  $\varepsilon$  and  $\delta$ , which can be alternatively used and has a simpler interpretation in terms of symmetries. These parameters are defined as

$$\varepsilon \equiv \frac{\varepsilon_1 + \varepsilon_2}{2}, \quad \delta \equiv \varepsilon_1 - \varepsilon_2 . \quad (5)$$

Their explicit expression in terms of the effective hamiltonian matrix elements, when CPT violation is introduced perturbatively and we may neglect terms which are quadratic in  $\Delta \equiv H_{22} - H_{11}$ , is given by

$$\varepsilon = \frac{\sqrt{H_{12}CP_{12}^*} - \sqrt{H_{21}CP_{12}}}{\sqrt{H_{12}CP_{12}^*} + \sqrt{H_{21}CP_{12}}}, \quad \delta = \frac{-2\Delta}{(\sqrt{H_{12}CP_{12}^*} + \sqrt{H_{21}CP_{12}})^2}. \quad (6)$$

The rephasing invariance<sup>3</sup> of  $\varepsilon$  and  $\delta$  is apparent from Eq. (6). In the following we will only keep linear terms in  $\delta$ .

When we pay attention to the restrictions imposed by discrete symmetries on the effective mass matrix,  $H = M - \frac{i}{2}\Gamma$ , we see that:

- CP conservation imposes  $\text{Im}(M_{12}\text{CP}_{12}^*) = \text{Im}(\Gamma_{12}\text{CP}_{12}^*) = 0$  and  $H_{11} = H_{22}$ ;
- CPT invariance requires  $H_{11} = H_{22}$ ;
- T invariance imposes  $\text{Im}(M_{12}\text{CP}_{12}^*) = \text{Im}(\Gamma_{12}\text{CP}_{12}^*) = 0$ .

As a consequence, CPT invariance leads to  $\Delta = 0$  and thus  $\delta = 0$ , irrespective of the value of  $\varepsilon$ . Similarly, T invariance leads to  $\varepsilon = 0$ , independently of the value of  $\delta$ . CP conservation requires both  $\varepsilon = \delta = 0$ . Therefore we have four real parameters which carry information on the symmetries of the effective mass matrix, according to the following list

- $\text{Re}(\varepsilon) \neq 0$  signals CP and T violation, with  $\Delta\Gamma \neq 0$ ;
- $\text{Im}(\varepsilon) \neq 0$  indicates CP and T violation;
- $\text{Re}(\delta) \neq 0$  means that CP and CPT violation exist;
- $\text{Im}(\delta) \neq 0$  shows CP and CPT violation, with  $\Delta\Gamma \neq 0$ .

To extract information on these symmetry parameters we may study the time evolution of the entangled state (2). As can be found in the literature, the special features of this system can be used to extract information on CP [9] and CPT [10] violation in  $B$  mesons.

We use for the final state the notation  $(X, Y)$ , where  $X$  is the decay product observed with momentum  $\vec{k}$  at a time  $t_0$ , and  $Y$  the product detected at a later time  $t$  with momentum  $-\vec{k}$ . The time variables  $\Delta t = t - t_0$  (with definite positive sign) and  $t' = t_0 + t$  are used to describe the process instead of  $t_0$  and  $t$ . The probability to find an arbitrary final state  $(X, Y)$  from the initial state (2) is given by

$$\begin{aligned}
|(X, Y)|^2 = & \frac{1}{8} \left( \frac{1 + |\varepsilon_1|^2}{1 - \varepsilon_1 \varepsilon_2} \right)^2 \left| \frac{1 + \varepsilon_2}{1 + \varepsilon_1} \right|^2 |\langle X|B_1\rangle|^2 |\langle Y|B_1\rangle|^2 e^{-\Gamma t'} \times \\
& \times \left\{ (\eta_+ + \eta_-) \cosh\left(\frac{\Delta\Gamma\Delta t}{2}\right) - (\eta_+ - \eta_-) \cos(\Delta m\Delta t) \right. \\
& \left. + 2\eta_{\text{re}} \sinh\left(\frac{\Delta\Gamma\Delta t}{2}\right) - 2\eta_{\text{im}} \sin(\Delta m\Delta t) \right\}, \tag{7}
\end{aligned}$$

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<sup>3</sup>We are using the notation  $H_{ij}$ ,  $\text{CP}_{ij}$ , etc. to represent the matrix elements of the corresponding operators in the flavour basis, for instance  $H_{12} \equiv \langle B^0|H|\bar{B}^0\rangle$ .

where  $\Gamma$  is the averaged width of the physical states  $B_{1,2}$ . The  $\eta$  coefficients are defined as

$$\begin{aligned}\eta_+ &= |\eta_X + \eta_Y|^2, \\ \eta_- &= |\eta_X - \eta_Y|^2, \\ \eta_{\text{re}} &= \text{Re}[(\eta_X + \eta_Y)(\eta_X^* - \eta_Y^*)], \\ \eta_{\text{im}} &= \text{Im}[(\eta_X + \eta_Y)(\eta_X^* - \eta_Y^*)],\end{aligned}\tag{8}$$

with

$$\eta_X \equiv \frac{\langle X|B^0\rangle - \frac{1-\varepsilon_2}{1+\varepsilon_2}\text{CP}_{12}^*\langle X|\bar{B}^0\rangle}{\langle X|B^0\rangle + \frac{1-\varepsilon_1}{1+\varepsilon_1}\text{CP}_{12}^*\langle X|\bar{B}^0\rangle} = \frac{1+\varepsilon_1}{1+\varepsilon_2} \cdot \frac{\varepsilon_2\langle X|B_+\rangle + \langle X|B_-\rangle}{\langle X|B_+\rangle + \varepsilon_1\langle X|B_-\rangle},\tag{9}$$

and an analogous expression for  $\eta_Y$ . We write Eq. (9) in terms of both flavour and CP eigenstates. From the above expressions one can easily check that for  $X = Y$  only  $\eta_+$  remains and the probability  $|(X, Y)|^2$  vanishes for  $\Delta t = 0$ . This is a kind of EPR correlation, imposed here by Bose statistics [11]. The integration of the probability (7) over  $t'$  between  $\Delta t$  (always positive with our definition) and  $\infty$  gives the intensity for the chosen final state, depending only on  $\Delta t$

$$I(X, Y; \Delta t) = \frac{1}{2} \int_{\Delta t}^{\infty} dt' |(X, Y)|^2.\tag{10}$$

By comparing the intensities corresponding to different processes one builds time-dependent asymmetries that allow the extraction of the relevant parameters.

### 3 Flavour Tag: Genuine observables needing $\Delta\Gamma$

As we mentioned in the introduction, when a semileptonic final state is seen at one side of the detector, the surviving meson can be tagged as the conjugated flavour state. Its later decay through another semileptonic channel is equivalent to the projection of the evolved meson onto the corresponding definite flavour state. In our notation, we represent as  $\ell^\pm$  the final decay product of a semiinclusive decay  $B \rightarrow \ell^\pm X^\mp$ . Thus, the final configuration we denote  $(\ell, \ell)$  is equivalent to a *flavour*  $\rightarrow$  *flavour* evolution, at the meson level. In Table 1 we show the equivalence for the four possible configurations with two charged leptons in the final state.

Transition	$\bar{B}^0 \rightarrow B^0$	$B^0 \rightarrow \bar{B}^0$	$\bar{B}^0 \rightarrow \bar{B}^0$	$B^0 \rightarrow B^0$
$(X, Y)$	$(\ell^+, \ell^+)$	$(\ell^-, \ell^-)$	$(\ell^+, \ell^-)$	$(\ell^-, \ell^+)$

Table 1: Meson transitions corresponding to semileptonic  $(X, Y)$  final decay products

From the processes of Table 1 we may construct two non-trivial asymmetries. If we compare the intensities for the first and second processes and suppose there is no violation either of the

$\Delta B = \Delta Q$  rule or of CPT in the direct decay amplitudes, we obtain the asymmetry

$$A(\ell^+, \ell^+) \equiv \frac{I(\ell^+, \ell^+) - I(\ell^-, \ell^-)}{I(\ell^+, \ell^+) + I(\ell^-, \ell^-)} = \frac{1 - \left| \frac{1-\varepsilon_2}{1+\varepsilon_2} \right|^2 \left| \frac{1-\varepsilon_1}{1+\varepsilon_1} \right|^2}{1 + \left| \frac{1-\varepsilon_2}{1+\varepsilon_2} \right|^2 \left| \frac{1-\varepsilon_1}{1+\varepsilon_1} \right|^2} \approx \frac{4 \frac{\text{Re}(\varepsilon)}{1+|\varepsilon|^2}}{1 + 4 \left( \frac{\text{Re}(\varepsilon)}{1+|\varepsilon|^2} \right)^2}, \quad (11)$$

where only linear terms in  $\delta$  have been kept, since CPT violation is treated in perturbation theory. We observe that this Kabir asymmetry [1] is time independent.

This is a genuine CP and T asymmetry, since the second process corresponds to the CP-, or T-transformed of the first one. Thus the asymmetry cannot be faked by absorptive parts in absence of true T violation. However, in the limit  $\Delta\Gamma = 0$  both  $\text{Re}(\varepsilon)$  and  $\text{Im}(\delta)$  vanish, and this quantity will be zero, even if CP and T violation exist. So this observable, in spite of being CP- and T-odd, and constituting a true observable of CP and T violation, also needs, in order to be non-zero, the presence of  $\Delta\Gamma \neq 0$ . This requirement is fulfilled in the kaon system, where the asymmetry (11) has actually been measured [2]. On the contrary, for  $B_d$ -mesons the negligible value of  $\Delta\Gamma$  predicts that this asymmetry will be difficult to observe. Present limits for  $\text{Re}(\varepsilon)$  are at the level of few percent [12].

The second asymmetry to be constructed from semileptonic decays is [13]

$$\begin{aligned} A(\ell^+, \ell^-) &\equiv \frac{I(\ell^+, \ell^-) - I(\ell^-, \ell^+)}{I(\ell^+, \ell^-) + I(\ell^-, \ell^+)} \\ &\approx -2 \frac{\text{Re}\left(\frac{\delta}{1-\varepsilon^2}\right) \text{Sh}\frac{\Delta\Gamma\Delta t}{2} - \text{Im}\left(\frac{\delta}{1-\varepsilon^2}\right) \sin(\Delta m\Delta t)}{\text{Ch}\frac{\Delta\Gamma\Delta t}{2} + \cos(\Delta m\Delta t)}, \end{aligned} \quad (12)$$

also to linear order in  $\delta$ . This asymmetry, contrary to that in Eq. (11), depends on time as an odd function of  $\Delta t$ . Eq. (12) corresponds to a genuine CP and CPT asymmetry, as it compares the probabilities for  $B^0 \rightarrow B^0$  and  $\bar{B}^0 \rightarrow \bar{B}^0$ . These transitions are self-conjugated under time reversal transformation, so T violation is not operative in this case. To get a non-zero value of the asymmetry (12), both CP and CPT violation are required, but also  $\Delta\Gamma \neq 0$ . The proportionality to  $\Delta\Gamma$  is present in both terms of the asymmetry, explicitly in the first one and in the second term through  $\text{Im}(\delta)$ . Therefore measuring a small limit for this observable does not give a straightforward bound on CPT violation, because the vanishingly small  $\Delta\Gamma$  of  $B$ -mesons would hide any symmetry breaking effect. Present limits [12] on  $\text{Im}(\delta)$  are again at the level of few percent.

## 4 CP Tag: Genuine observables which do not need $\Delta\Gamma$

As explicitly seen in Eqs. (11) and (12), the flavour asymmetries are difficult to prove discrete symmetry violation in the  $B$ -system, due to the negligible  $\Delta\Gamma$  between the physical states. However, we may construct alternative asymmetries making use of the CP eigenstates, which can be identified in this system by means of a CP tag.

From the entangled state (3) such a tag can be performed. Let us suppose that  $X$ , a CP eigenstate produced along the CP-conserving direction, is observed at  $t_0$  at one side of the detector. Such a decay is free from direct CP violation, and this assures that in the moment of the decay the surviving meson of the other side had the opposite CP eigenvalue. One example of a decay channel with the properties we are looking for is given by  $X \equiv J/\Psi K_S$ , with  $\text{CP} = -$ , or by  $X \equiv J/\Psi K_L$ , with  $\text{CP} = +$ , both of them governed by the “CP-allowed” side  $V_{cb}V_{cs}^*$  of the  $(bs)$  triangle. Then, the detection of such a final state leads to the preparation of the remaining  $B_d$  meson in the complementary CP eigenstate, to  $\mathcal{O}(\lambda^3)$ . Since CP is conserved in  $K$  and  $B_s$  systems to  $\mathcal{O}(\lambda^3)$ , it is correct to identify the physical kaon states with the CP eigenstates, at least to this order in the expansion.

After this preparation, the CP tagged state is left to evolve for a certain time  $\Delta t$ . We are interested in the transition probabilities from CP-eigenstates,  $B_+$  and  $B_-$ , to flavour states,  $B^0$  and  $\bar{B}^0$ . Thus we will be looking at final configurations such as  $(J/\Psi K_S, \ell^+)$  that corresponds, at the meson level, to  $B_+ \rightarrow B^0$ . All the transitions connected to this one by symmetry transformations, are shown in Table 2.

Transition	$B_+ \rightarrow \bar{B}^0$	$\bar{B}^0 \rightarrow B_+$	$B^0 \rightarrow B_+$
$(X, Y)$	$(J/\Psi K_S, \ell^-)$	$(\ell^+, J/\Psi K_L)$	$(\ell^-, J/\Psi K_L)$
Transformation	CP	CPT	T

Table 2: Transitions and  $(X, Y)$  final configurations connected to  $B_+ \rightarrow B^0$  and  $(J/\Psi K_S, \ell^+)$  by symmetry transformations.

Comparing the intensity of  $(J/\Psi K_S, \ell^+)$  with each of the processes on Table 2, we construct three genuine asymmetries of the form

$$A(X, Y) = \frac{I(X, Y) - I(J/\Psi K_S, \ell^+)}{I(X, Y) + I(J/\Psi K_S, \ell^+)}. \quad (13)$$

Each of them is odd under the discrete transformation which connects  $(J/\Psi K_S, \ell^+)$  with  $(X, Y)$ . To linear order in  $\delta$  and in the limit  $\Delta\Gamma = 0$ , we get the following results:

- The CP odd asymmetry,

$$A_{\text{CP}} \equiv A(J/\Psi K_S, \ell^-) = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) + \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right), \quad (14)$$

has contributions from T-violating and CPT-violating terms. The first term, odd in  $\Delta t$ , is governed by the T-violating  $\text{Im}(\varepsilon)$ , whereas the second term, which is even in  $\Delta t$ , is sensitive to CPT violation through the parameter  $\text{Re}(\delta)$ . The CP asymmetry corresponds to the well known “gold plate” decay [14] and has been measured by CDF [15]. The inclusion of CPT violation through  $\text{Re}(\delta)$  has been considered previously in Ref. [13]. The



equivalence between final configurations and mesonic transitions allows the interpretation of this observable in terms of *CP-to-flavour* transitions. It constitutes then a test of indirect CP, either by T violation or by CPT violation. The separation of T-odd and CPT-odd terms can be done by constructing different asymmetries, which are discussed in the following.

- The T asymmetry,

$$A_T \equiv A(\ell^-, J/\Psi K_L) = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right], \quad (15)$$

purely odd in  $\Delta t$ , needs  $\varepsilon \neq 0$ , and includes CPT even and odd terms.

- The CPT asymmetry,

$$A_{\text{CPT}} \equiv A(\ell^+, J/\Psi K_L) = \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \frac{1}{1 - 2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t)} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right), \quad (16)$$

needs  $\delta \neq 0$ , and includes both even and odd time dependences, so that there is no definite symmetry under a change of sign of  $\Delta t$ .

There is still a fourth discrete transformation, consisting in the exchange in the order of appearance of the decay products  $X$  and  $Y$ , i. e.  $\Delta t \rightarrow -\Delta t$ . It transforms

$$\begin{aligned} B_+ \rightarrow B^0 \\ (J/\Psi K_S, \ell^+) \end{aligned} \xrightarrow{\Delta t} \begin{aligned} \bar{B}^0 \rightarrow B_- \\ (\ell^+, J/\Psi K_S) \end{aligned} . \quad (17)$$

Looking at its effect at the meson level it turns out that  $\Delta t$  reversal cannot be associated to any of the fundamental discrete symmetries. In spite of this, it can still provide information on the symmetries of the system. In the limit  $\Delta\Gamma = 0$ , the temporal asymmetry satisfies

$$A_{\Delta t} \equiv A(\ell^+, J/\Psi K_S) = A(\ell^-, J/\Psi K_L) \equiv A_T . \quad (18)$$

This equality is a consequence of  $\Delta\Gamma = 0$ . In general, the equivalence of T and  $\Delta t$  inversions is only valid for hamiltonians with the property of hermiticity, up to a global (proportional to unity) absorptive part. For the kaon system, for instance, relation (18) does not hold.

Measuring the presented asymmetries (14)-(18) with good time resolution, so to separate even and odd  $\Delta t$  dependences, should be enough to determine the parameters

$$\frac{2\text{Im}(\varepsilon)}{1 + |\varepsilon|^2}, \quad \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2}, \quad (19)$$

governing CP, T violation and CP, CPT violation, respectively, for the  $B_d$  mixing.

Contrary to what happened in section 3, the CPT and T asymmetries based on a CP tag do not vanish due to the smallness of  $\Delta\Gamma$ . Instead, they provide a set of observables which could separate the parameters  $\delta$  and  $\varepsilon$ .

In case of CPT invariance,  $\delta = 0$ ,  $A_{\text{CPT}} = 0$ , and the four asymmetries reduce to a single independent one  $A_{\text{CP}} = A_{\text{T}} = A_{\Delta t}$ . In the Standard Model this measures  $\sin(2\beta)$  of the  $(bd)$  unitarity triangle [16]. We emphasize, however, that one has access to four experimentally different asymmetries whose results would also be different in case of CPT violation. In extended theories, efforts to find a plausible framework for CPT violation are discussed in Ref. [17].

## 5 Non-genuine observables only with $J/\Psi K_S$

All the asymmetries defined in the previous section are genuine observables, since each of them compares the original process with its conjugated under a certain symmetry and is thus odd under the corresponding transformation. Nevertheless the measurement of all those quantities requires to tag both  $B_+$  and  $B_-$  states. The last needs, from the experimental point of view, a good reconstruction of the decay  $B \rightarrow J/\Psi K_L$ , not so easy to achieve as for the corresponding  $J/\Psi K_S$  channel. Therefore we may wonder how much can be learned about the symmetry parameters from the study of all possible asymmetries built from final configurations  $(X, Y)$  with only  $J/\Psi K_S$ .

We show that if one considers only final states in which a  $K_S$  is present, the same asymmetries can still be constructed, provided that the limit  $\Delta\Gamma = 0$  is valid. In this case there are four possible configurations of the final state, depending on the sign of the charged lepton and on the order of appearance of the decay products,

$$(J/\Psi K_S, \ell^+), \quad (J/\Psi K_S, \ell^-), \quad (\ell^+, J/\Psi K_S), \quad (\ell^-, J/\Psi K_S). \quad (20)$$

We show in Table 3 the mesonic transitions which are related to each final state.

Transition	$B_+ \rightarrow B^0$	$B_+ \rightarrow \bar{B}^0$	$\bar{B}^0 \rightarrow B_-$	$B^0 \rightarrow B_-$
$(X, Y)$	$(J/\Psi K_S, \ell^+)$	$(J/\Psi K_S, \ell^-)$	$(\ell^+, J/\Psi K_S)$	$(\ell^-, J/\Psi K_S)$

Table 3:  $(X, Y)$  configurations with  $J/\Psi K_S$  and the corresponding meson transitions

In the exact limit  $\Delta\Gamma = 0$ , taking into account that  $\Delta t$  and T operations become equivalent, one can construct asymmetries which measure indirect violation of CP, T and CPT from the processes shown in Table 3.

Thus, we obtain, to linear order in the CPT violating parameter  $\delta$ ,

•

$$A(J/\Psi K_S, \ell^-) = 2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) + 2 \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{\text{Re}(\delta)}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right). \quad (21)$$

This is the asymmetry  $A_{\text{CP}}$  defined in Eq. (14). It measures  $\text{Re}(\delta)$  or  $\text{Im}(\varepsilon)$  different from zero, and constitutes indeed a genuine measurement of indirect CP violation. Since it

contains  $\Delta t$  odd and  $\Delta t$  even pieces, both parameters could be separated. An equivalent observable would be given by the asymmetry between the third and fourth processes of Table 3, also related by a CP transformation.

•

$$A(\ell^+, J/\Psi K_S) = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - 2 \frac{\text{Re}(\delta)}{1 + |\varepsilon|^2} \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right]. \quad (22)$$

This is the temporal asymmetry defined in the previous section. It is only different from zero if  $\text{Im}(\varepsilon)$  is not null. Then it measures indirect violation of T when  $\Delta\Gamma = 0$ . The second and fourth decays of the table are connected by a  $\Delta t$  exchange, too, and will measure the same parameters.

•

$$A(\ell^-, J/\Psi K_S) = 2 \frac{\text{Re}(\delta)}{1 + |\varepsilon|^2} \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \frac{1}{1 - 2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t)}. \quad (23)$$

This quantity is different from zero only if  $\text{Re}(\delta)$  is not null. It equals the CPT asymmetry  $A_{\text{CPT}}$  in Eq. 16 and can be used to measure CPT violation if  $\Delta\Gamma = 0$ .

These results point out a way of testing the discrete symmetries in the evolution of  $B$ -system, from data of entangled states decays. The observed events where a  $K_S$  is present in the final decay products can be classified according to Eq.(20). This is enough to construct the different asymmetries sensitive to CP, T and CPT violating parameters  $\text{Im}(\varepsilon)$  and  $\text{Re}(\delta)$ , as long as  $\Delta\Gamma = 0$ . The method exploits the good  $\Delta t$ -time resolution accessible in asymmetric  $B$ -factories. The separation of  $\varepsilon$  and  $\delta$  is associated with resolving odd and even functions of  $\Delta t$ , respectively.

## 6 Linear $\Delta\Gamma$ corrections

The asymmetries in Eqs. (22) and (23), contrary to those built in section 4, are not genuine. They do not correspond to true T- and CPT-odd observables, for the processes we are comparing are not related by a symmetry transformation. This implies that the presence of  $\Delta\Gamma \neq 0$  may induce non-vanishing values of the asymmetries discussed in section 5, even if there is no true T- or CPT-violation.

Although in the limit  $\Delta\Gamma = 0$  the results of Eqs. (22) and (23) agree, respectively, with those in Eqs. (15) and (16), experimentally one is measuring different quantities. When  $\Delta\Gamma \neq 0$  is considered, absorptive parts affect all the above expressions, giving corrections to both genuine and non-genuine asymmetries. These absorptive effects appear differently in observables which yielded the same result before, and originate fake contributions to non-genuine asymmetries. To analyze them, we must take into account two types of corrections:

1. time-dependent terms in the amplitudes which are governed by the evolution  $\frac{\Delta\Gamma\Delta t}{2}$ ,

2. non-vanishing values of  $\text{Re}(\varepsilon)$  and  $\text{Im}(\delta)$ .

To treat the problem in a systematic way, it is convenient to study how the parameters  $\text{Re}(\varepsilon)$  and  $\text{Im}(\delta)$  depend on the non-vanishing value of  $\Delta\Gamma$ .

In terms of the matrix elements of the effective hamiltonian, the relevant parameters were given by Eq. (6). From these expressions we see

$$\text{Re}(\varepsilon) \propto |M_{12}|^2 \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right), \quad \text{Im}(\delta) \propto \Delta |M_{12}| \left|\frac{\Gamma_{12}}{M_{12}}\right|. \quad (24)$$

Since  $\left|\frac{\Gamma_{12}}{M_{12}}\right|$  is proportional to  $\Delta\Gamma$ , and we are only interested in the first corrections to our asymmetries, we may neglect  $\text{Im}(\delta)$ , which is already first order in both  $\Delta\Gamma$  and  $\Delta$ , and parametrize

$$\varepsilon \equiv x\Delta\Gamma + i\text{Im}(\varepsilon), \quad \delta \equiv \text{Re}(\delta). \quad (25)$$

Using this parametrization it is now easy to throw away terms of order higher than one in  $\Delta\Gamma$  or  $\text{Re}(\delta)$ . The parameter  $\text{Im}(\varepsilon)$  will have also  $\Delta\Gamma$  corrections. However, the new terms will accompany the zero order  $\text{Im}(\varepsilon)$  and have the same time dependence, so that we will be unable to separate them. Thus an explicit expansion of  $\text{Im}(\varepsilon)$  is not needed for our study.

## 6.1 Corrections to flavour observables

We discussed in section 3 two asymmetries that could be built from flavour tag, Eq. (11) and Eq. (12). Both of them vanished in case of  $\Delta\Gamma = 0$ . If we include linear  $\Delta\Gamma$  corrections, as described above, the first observable,  $A(\ell^+, \ell^-)$ , acquires a non zero value

$$A(\ell^+, \ell^+) \approx \frac{4x\Delta\Gamma}{1 + |\varepsilon|^2}, \quad (26)$$

whereas the second one,  $A(\ell^+, \ell^-)$ , is linear in both  $\Delta\Gamma$  and  $\Delta$ , and vanishes also at this order.

## 6.2 Corrections to genuine *CP-to-flavour* observables

For the genuine asymmetries of section 4, based on a CP tag, we get

•

$$\begin{aligned} A_{\text{CP}} &= \frac{I(J/\Psi K_S, \ell^+) - I(J/\Psi K_S, \ell^-)}{I(J/\Psi K_S, \ell^+) + I(J/\Psi K_S, \ell^-)} \\ &\simeq \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \sin^2\left(\frac{\Delta m \Delta t}{2}\right) - \frac{2\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \end{aligned}$$

$$\begin{aligned}
& + \frac{4x\Delta\Gamma}{1+|\varepsilon|^2} \left[ 1 + 2 \left( \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \right)^2 \right] \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \\
& - \frac{\Delta\Gamma\Delta t}{2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) - \frac{8\Delta\Gamma x}{1+|\varepsilon|^2} \left( \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \right)^2 \sin^4 \left( \frac{\Delta m \Delta t}{2} \right) \quad (27)
\end{aligned}$$

With respect to Eq.(14), the  $\Delta\Gamma$  corrections induce both  $\Delta t$  even and odd functions. The first two terms in Eq.(27) reproduce the result for  $\Delta\Gamma = 0$ , whereas the third term has the same time dependence of the first one and would make it difficult to extract  $\text{Re}(\delta)$  from this CP asymmetry. The last two terms have new  $\Delta t$  dependences.

•

$$\begin{aligned}
A_T &= \frac{I(J/\Psi K_S, \ell^+) - I(\ell^-, J/\Psi K_L)}{I(J/\Psi K_S, \ell^+) + I(\ell^-, J/\Psi K_L)} \\
&\simeq -\frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right] \\
&+ \frac{4x\Delta\Gamma}{1+|\varepsilon|^2} \left[ 1 - 2 \left( \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \right)^2 \right] \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \\
&+ \frac{\Delta\Gamma\Delta t}{2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) + \frac{8x\Delta\Gamma}{1+|\varepsilon|^2} \left( \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \right)^2 \sin^4 \left( \frac{\Delta m \Delta t}{2} \right) \quad (28)
\end{aligned}$$

Contrary to what happens for  $A_{\text{CP}}$ , all the new terms in  $A_T$  from linear  $\Delta\Gamma$  corrections have different  $\Delta t$  dependences as compared to those in Eq.(15).

•

$$\begin{aligned}
A_{\text{CPT}} &= \frac{I(J/\Psi K_S, \ell^+) - I(\ell^+, J/\Psi K_L)}{I(J/\Psi K_S, \ell^+) + I(\ell^+, J/\Psi K_L)} \\
&\simeq -\frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \frac{1}{1 - \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t)}. \quad (29)
\end{aligned}$$

To the order considered in our perturbation expansion,  $A_{\text{CPT}}$  has no linear  $\Delta\Gamma$  corrections. The genuine character of the asymmetry would put them in higher order terms. We conclude that, in presence of  $\Delta\Gamma$  corrections, the extraction of  $\text{Re}(\delta)$  should be done from  $A_{\text{CPT}}$ .

•

$$\begin{aligned}
A_{\Delta t} &= \frac{I(J/\Psi K_S, \ell^+) - I(\ell^+, J/\Psi K_S)}{I(J/\Psi K_S, \ell^+) + I(\ell^+, J/\Psi K_S)} \\
&\simeq -2 \frac{\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - 2 \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right] \\
&+ \frac{\Delta\Gamma\Delta t}{2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \\
&- \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - 2 \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right]. \quad (30)
\end{aligned}$$

From this equation we observe that the “equivalence” between time reversal and  $\Delta t$  exchange does not hold any longer. On the contrary, the difference between both asymmetries is now linear in  $\Delta\Gamma$ .

Except for  $A_{CPT}$  in Eq. (29), all the asymmetries have linear corrections in  $\Delta\Gamma$ . These terms make the analysis more complicated, but do not avoid the separation of parameters, provided one has enough statistics to identify the different time dependences in the asymmetries. It is worth paying closer attention to the way linear  $\Delta\Gamma$  corrections enter the different quantities.  $A_{CP}$ ,  $A_T$  and  $A_{CPT}$  are genuine symmetry observables, i. e. they are constructed from the comparison between processes related by a true symmetry transformation. They are in general affected by corrections in  $\Delta\Gamma$ , but these do not generate fake effects such that even in absence of symmetry violation a non-vanishing asymmetry remains due to  $\Delta\Gamma \neq 0$ . We can check that in expressions (27)-(29) all  $\Delta\Gamma$  factors are multiplied either by  $\text{Im}(\varepsilon)$  or by  $x$ , both vanishing in case of exact symmetries. On the contrary,  $A_{\Delta t}$ , which as we have discussed does not correspond to any discrete symmetry, contains terms in  $\Delta\Gamma$  when  $\varepsilon = \delta = 0$ .

### 6.3 Corrections to non-genuine *CP-to-flavour* observables

The fact that  $\Delta t$  and T inversions are no longer equivalent affects the non-genuine asymmetries we have built only with the  $J/\Psi K_S$  final state. If we study how the linear terms modify the expressions in Eqs. (21)-(23), we find

- the asymmetry in Eq. (21) is still a genuine CP asymmetry, and takes the same value as Eq. (27);
- the asymmetry of Eq. (22) is the temporal asymmetry, so it will be equal to that of Eq. (30), and cannot be identified with  $A_T$ , as was the case for  $\Delta\Gamma = 0$ ;
- in the same way, Eq. (23) is not equal to the true CPT asymmetry. Instead we get

$$A(\ell^-, J/\Psi K_S) \simeq \frac{1}{1 - \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t)} \left[ \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} + \frac{\Delta\Gamma \Delta t}{2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \right. \\ \left. + \frac{4x\Delta\Gamma}{1+|\varepsilon|^2} \sin^2\left(\frac{\Delta m \Delta t}{2}\right) - \frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) \right], \quad (31)$$

which contains linear terms in  $\Delta\Gamma$  even when  $\varepsilon = \delta = 0$ .

## 7 What can be seen in a symmetric factory?

In previous sections we saw the crucial role played by  $\Delta t$  resolution in order to separate the symmetry parameters of the  $B$ -system. To achieve the measurement of all the proposed

observables one needs to distinguish between configurations where final decay products  $X$  and  $Y$  occur with opposite time ordering, and also to determine the time dependence of the resulting quantities with good enough precision.

In symmetric  $e^+e^-$  facilities at the  $\Upsilon(4S)$  peak,  $B$  mesons are created at rest, so that they decay essentially at the point of production. Therefore the measurement has no time resolution, but we may still extract some information on indirect symmetry violation from the available observations.

## 7.1 Flavour Tag

If  $\Delta t$  is not measured, there are three different probabilities,

$$\begin{aligned}\Gamma_{++} &= \int_0^\infty d(\Delta t) I(\ell^+, \ell^+) = \frac{C}{4\Gamma} \left[ 1 + \frac{4x\Delta\Gamma}{1+|\varepsilon|^2} + \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \right] \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2}, \\ \Gamma_{--} &= \int_0^\infty d(\Delta t) I(\ell^-, \ell^-) = \frac{C}{4\Gamma} \left[ 1 - \frac{4x\Delta\Gamma}{1+|\varepsilon|^2} + \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \right] \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2}, \\ \Gamma_{+-} &= \int_0^\infty d(\Delta t) [I(\ell^+, \ell^-) + I(\ell^-, \ell^+)] = \frac{C}{2\Gamma} \left[ 1 + \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \right] \frac{2 + \left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2},\end{aligned}\quad (32)$$

where  $C$  is a common normalization factor for all the probabilities.

The only symmetry observable which can be constructed from these quantities is the asymmetry between  $\Gamma_{++}$  and  $\Gamma_{--}$ . This yields the same result of Eq. (11), since all time dependence factored out from  $I(\ell^+, \ell^+)$  and  $I(\ell^-, \ell^-)$ .

## 7.2 CP Tag

When we sum over all possible values of  $\Delta t$ , probabilities for channels which do not correspond to the same meson transition are added up. For instance,  $(J/\Psi K_S, \ell^+)$  and  $(\ell^+, J/\Psi K_S)$ , which correspond to  $B_+ \rightarrow B^0$  and  $\bar{B}^0 \rightarrow B_-$ , contribute to the same probability. In this way we may construct four quantities

$$\begin{aligned}\Gamma_{S+} &\equiv \int_0^\infty [I(J/\Psi K_S, \ell^+) + I(\ell^+, J/\Psi K_S)] d(\Delta t) \\ &= \frac{C}{\Gamma} \left\{ 1 + \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2} + \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \left[ 2 + \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2} \right] \right\}; \\ \Gamma_{S-} &\equiv \int_0^\infty [I(J/\Psi K_S, \ell^-) + I(\ell^-, J/\Psi K_S)] d(\Delta t) \\ &= \frac{C}{\Gamma} \left\{ 1 - \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2} + \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \left[ 2 - \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1 + \left(\frac{\Delta m}{\Gamma}\right)^2} \right] \right\};\end{aligned}$$

$$\begin{aligned}
\Gamma_{L+} &\equiv \int_0^\infty [I(J/\Psi K_L, \ell^+) + I(\ell^+, J/\Psi K_L)] d(\Delta t) = \\
&= \frac{C}{\Gamma} \left\{ 1 + \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2} + \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \left[ 2 - \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2} \right] \right\}; \\
\Gamma_{L-} &\equiv \int_0^\infty [I(J/\Psi K_L, \ell^-) + I(\ell^-, J/\Psi K_L)] d(\Delta t) = \\
&= \frac{C}{\Gamma} \left\{ 1 - \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2} + \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \left[ 2 + \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2} \right] \right\}. \quad (33)
\end{aligned}$$

Each one of these probabilities corresponds to a pair of meson transitions, according to Table 4. Nevertheless we may construct genuine asymmetries by comparing the probabilities

Probability	Transitions
$\Gamma_{S+}$	$B_+ \rightarrow B^0$ or $\bar{B}^0 \rightarrow B_-$
$\Gamma_{S-}$	$B_+ \rightarrow \bar{B}^0$ or $B^0 \rightarrow B_-$
$\Gamma_{L+}$	$B_- \rightarrow B^0$ or $\bar{B}^0 \rightarrow B_+$
$\Gamma_{L-}$	$B_- \rightarrow \bar{B}^0$ or $B^0 \rightarrow B_+$

Table 4: Mesonic processes associated to measurable probabilities without time resolution.

for conjugated pairs of transitions. In this way we find

$$\mathcal{A}_{\text{CP}} \equiv \frac{\Gamma_{S+} - \Gamma_{S-}}{\Gamma_{S+} + \Gamma_{S-}} \approx \left[ \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} + \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \right] \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2}, \quad (34)$$

$$\mathcal{A}_{\text{T}} \equiv \frac{\Gamma_{S+} - \Gamma_{L-}}{\Gamma_{S+} + \Gamma_{L-}} \approx \frac{2x\Delta\Gamma}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2}, \quad (35)$$

$$\mathcal{A}_{\text{CPT}} \equiv \frac{\Gamma_{S+} - \Gamma_{L+}}{\Gamma_{S+} + \Gamma_{L+}} \approx \frac{\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\left(\frac{\Delta m}{\Gamma}\right)^2}{1+\left(\frac{\Delta m}{\Gamma}\right)^2}. \quad (36)$$

Since no time dependence is measured, the distinction between genuine and non-genuine observables, which was based on the equivalence between  $\Delta t$  and T reversals for  $\Delta\Gamma = 0$ , does not apply in this case. All the asymmetries here are genuine, since none of them will give a non-zero value due to  $\Delta\Gamma$  corrections, unless there is also a violation of the corresponding symmetry. Looking at the expressions, we realize that  $\mathcal{A}_{\text{CPT}}$  is proportional to  $\text{Re}(\delta)$ , different from zero only if CPT is not conserved.  $\mathcal{A}_{\text{T}}$  is proportional to the T-violating  $\text{Re}(\varepsilon)$ , so that it needs also the presence of absorptive parts to be detected. Finally the CP asymmetry  $\mathcal{A}_{\text{CP}}$ , already considered in the literature [13], contains both a T-violating and a CPT-violating term, with no chance to separate  $\text{Re}(\varepsilon)$  and  $\text{Re}(\delta)$ . The most interesting result is therefore the access to a non-vanishing value of  $\text{Re}(\delta)$ , even for  $\Delta\Gamma = 0$ , from  $\mathcal{A}_{\text{CPT}}$ .



There is a constant factor  $\frac{(\frac{\Delta m}{\Gamma})^2}{1+(\frac{\Delta m}{\Gamma})^2}$  common to all the asymmetries. For the  $B_d$ -system  $\frac{\Delta m}{\Gamma} = 0.723 \pm 0.032$  [18], so that this factor will not dilute appreciably the value of the observables.

## 8 Conclusions

In this paper we have studied in detail the possibilities to explore indirect violation of discrete symmetries CP, T, CPT in a neutral meson system. We have shown how, even in absence of relative absorptive parts, i. e., if  $\Delta\Gamma = 0$ , this study is possible, but one needs observables beyond *flavour-to-flavour* transitions due to time evolution of the meson system. In addition to flavour tags, then, we have considered also CP tags, which are uniquely defined for the  $B_d$ -system at order  $\mathcal{O}(\lambda^3)$ , enough to include CP and T violation at leading order. Possible CPT violation is included perturbatively. This consistent scheme of the treatment of symmetry violation can be tested at the  $B$  factories, where the production of entangled states of  $B_d$  mesons allows the preparation of the meson in a CP eigenstate.

The asymmetries analyzed in this work exploit their time dependences in order to separate out two different ingredients. On one hand CP and T violation, described by  $\varepsilon$ , and on the other CP and CPT violation, given by  $\delta$ . The complex parameters  $\varepsilon$  and  $\delta$  are defined in a rephasing invariant way and, due to the well defined CP operator, they are unique and physical quantities.

The observables we have described can be classified in three types:

1. Genuine asymmetries for T or CPT violation, based on *flavour-to-flavour* transitions at the meson level, which need the support of absorptive parts,  $\Delta\Gamma \neq 0$ . The T asymmetry measures  $\frac{\text{Re}(\varepsilon)}{1+|\varepsilon|^2}$  and does not depend on time. The CPT asymmetry is odd in  $\Delta t$ , but involves  $\text{Re}(\frac{\delta}{1-\varepsilon^2})$  and  $\text{Im}(\frac{\delta}{1-\varepsilon^2})$  with different time dependences. Since both asymmetries are zero in absence of  $\Delta\Gamma$ , these are not promising observables for  $B_d$  physics.
2. Genuine observables which do not need  $\Delta\Gamma$ . We have constructed three different asymmetries, signals of CP, CPT and T violation, respectively, which are based on the combination of flavour and CP tags. In the limit  $\Delta\Gamma = 0$  they involve  $\frac{\text{Im}(\varepsilon)}{1+|\varepsilon|^2}$  and  $\frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{\text{Re}(\delta)}{1+|\varepsilon|^2}$ , which can be separated out from the different time dependences. There is yet a fourth quantity, the temporal asymmetry, experimentally different from the others, which becomes theoretically equivalent to time reversal asymmetry only in the limit  $\Delta\Gamma = 0$ . In practice, this kind of observables needs the identification of a semileptonic decay in one side and  $J/\Psi K_S$ ,  $J/\Psi K_L$  decays in the other side.
3. Making use of the equivalence between  $\Delta t$  and T reversal operations for  $\Delta\Gamma = 0$ , we have also considered a group of non genuine observables. They are equivalent to the genuine observables described in the previous paragraph in the limit  $\Delta\Gamma = 0$ , with the advantage

that they involve only the hadronic decay  $J/\Psi K_S$ . As a consequence, these asymmetries are accessible in the first generation of  $B$  factory experimental results. These observables are non genuine in the sense that absorptive parts can mimic a non-vanishing value for them, even if violation of the fundamental symmetry is not present.

The different character of genuine and non genuine observables becomes apparent when corrections due to a non zero  $\Delta\Gamma$  are included. While in genuine observables the effect of  $\Delta\Gamma$  is limited to correct the existing terms of  $\varepsilon$  and  $\delta$ , non genuine asymmetries acquire new terms, proportional to  $\Delta\Gamma$ , even for  $\varepsilon = \delta = 0$ .

In a symmetric  $B$  factory, all the effects which are odd in  $\Delta t$  are automatically cancelled by the integration over time implicit in all the experimentally measured quantities. Nevertheless we have seen that some relevant information survives, associated with even terms in  $\Delta t$ .

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